Minitest 1 - MTH 2410

Dr. Graham-Squire, Fall 2012

8:38
8:55

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are not allowed on this test. Calculators are allowed on all parts of the test, however you should still show all of your work.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 7. Total Points = 45.

1. (6 points) Let $\mathbf{u} = \langle 0, 2, 3 \rangle$, $\mathbf{v} = \langle -1, 0, 4 \rangle$ and $\mathbf{w} = \langle 2, -3, 0 \rangle$. Calculate the following expressions. If the expression does not exist or does not make sense, explain why.

(a) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$

$$\vec{u}\times\vec{v} = \begin{array}{ccccc} i & j & k & i & j \\ 0 & 2 & 3 & 0 & 2 \\ -1 & 0 & 4 & -1 & 0 \end{array} = (8-0)\vec{i} + (-3-0)\vec{j} + (0-(-2))\vec{k}$$

$$= \langle 8, -3, 2 \rangle$$

$$(\vec{u}\times\vec{v})\times\vec{w} = \begin{array}{ccc} i & j & k \\ 8 & -3 & 2 \\ 2 & -3 & 0 \end{array} = 6\vec{i} + (4-0)\vec{j} + (-24+6)\vec{k}$$

$$= \boxed{\langle 6, 4, -18 \rangle}$$

(b) $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$

does not exist because $u \cdot v$ is a scalar and you can't dot a scalar with a vector.

2. (8 points) TRUE OR FALSE. Circle the correct answer. If false, give a counterexample or explain (briefly) why it is false.

(a) **True or False:** For any vectors $\mathbf{c}$ and $\mathbf{d}$, we find $\mathbf{c} \cdot \mathbf{d} = \mathbf{d} \cdot \mathbf{c}$.

True.

(b) **True or False:** For any space vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

False $u \times v = -v \times u$

(c) **True or False:** If $\mathbf{a} \cdot \mathbf{b} = 0$ then either $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = \mathbf{0}$.

False $\langle 1, 0 \rangle \cdot \langle 0, 1 \rangle = 0 + 0 = 0$

but $\vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{0}$

(d) **True or False:** Orthogonal planes can be described using the same normal vector.

False. Orthogonal planes have orthogonal normal vectors.

3. (4 points) Find the distance between the point (1,8,5) $= Q$ and the plane $2x + y - z = 5$.

$\vec{n}$ = normal vector to plane $= \langle 2, 1, -1 \rangle$ ✓

point on plane $P = \langle 0, 5, 0 \rangle$ ✓

$\overrightarrow{PQ} = \langle 1, 3, 5 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \vec{n}|}{\|n\|} = \frac{|2+3-5|}{\sqrt{6}} = \frac{0}{\sqrt{6}} = \boxed{0}$$

✓✓

4. (4 points) Write a set of parametric equations that represent a line through the points (0,4,3) and (-1,2,5).

P Q

$\overrightarrow{PQ} = \langle -1, -2, 2 \rangle$ = direction vector for line ✓✓

$-1 = a$, $-2 = b$, $2 = c$

$(0, 4, 3)$

$0 = x_1$, $4 = y_1$, $3 = z_1$

$$\boxed{x = -t, \quad y = 4 - 2t, \quad z = 3 + 2t}$$

✓

or

$x = -1 - t, \quad y = 2 - 2t, \quad z = 5 + 2t$

5. (6 points) Convert the given point or surface to the coordinate system specified.

(a) The surface $r^2 + z^2 - 9z = 0$ in cylindrical coordinates to spherical coordinates.

$\cancel{\rho^2 \sin^2(\phi) \ldots z}$

$\rho = \sqrt{r^2+z^2}$, $z = \rho\cos(\phi)$

$\Rightarrow$ surface is $\dfrac{\rho^2 - 9\rho\cos(\phi)}{\rho} = \dfrac{0}{\rho}$

$\rho - 9\cos(\phi) = 0$

$\boxed{\rho = 9\cos(\phi)}$

(b) Spherical coordinates $(12, \frac{\pi}{2}, 0)$ to Cartesian coordinates. $\Leftarrow$ rectangular

$\cancel{r^2 = 12^2 \sin^2(0) \quad \theta = \frac{\pi}{2}}$,

$x = 12\sin(0)\cos(\frac{\pi}{2}) = 0$

$y = 12\sin(0)\sin(\frac{\pi}{2}) = 0$

$z = 12\cos(0) = 12$

$\boxed{(0,0,12)}$

No Calculator Name: Key

6. (10 points) Match the equation to the graph.

(iii) (a) $9x^2 - 6y^2 + 9z^2 = 36$

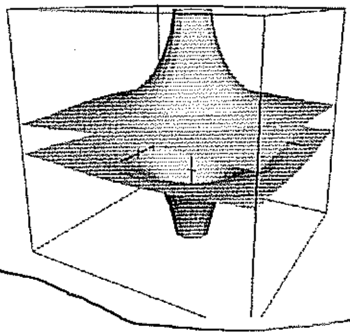
(iv) (b) $x^2 - 25y^2 = 9z^2 + 36$ $\Rightarrow$ $x^2 - 25y^2 - 9z = 36$

(ii) (c) $y^2 + z^2 = 25$ cylinder.

(v) (d) $10y = 10z^2 - x^2$

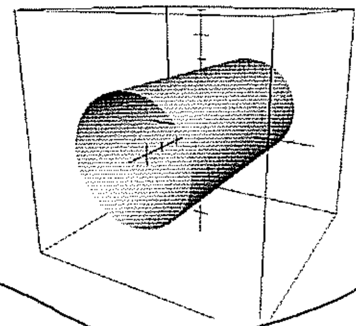
(i) (e) $x^2 + y^2 = \dfrac{1}{z^2}$ rotational surface with $r(z) = \frac{1}{z}$

(e)



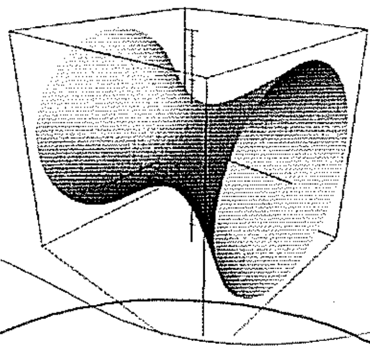
(i)

(c)



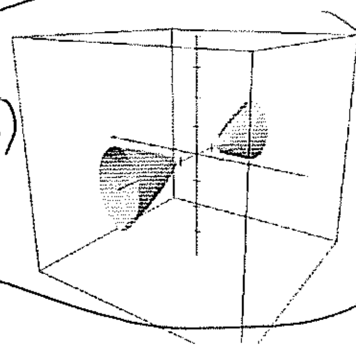
(ii)

(a)

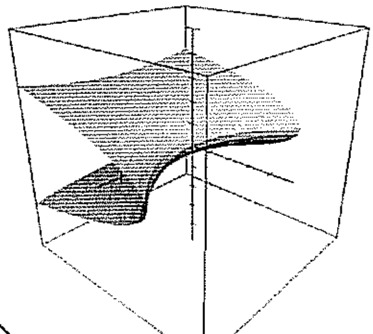


(iii)

(b)

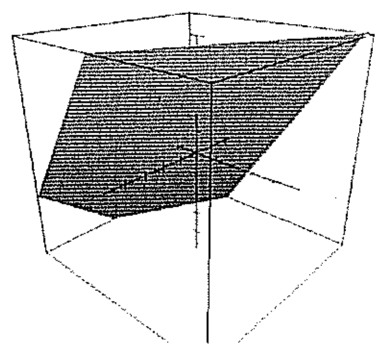


(iv)



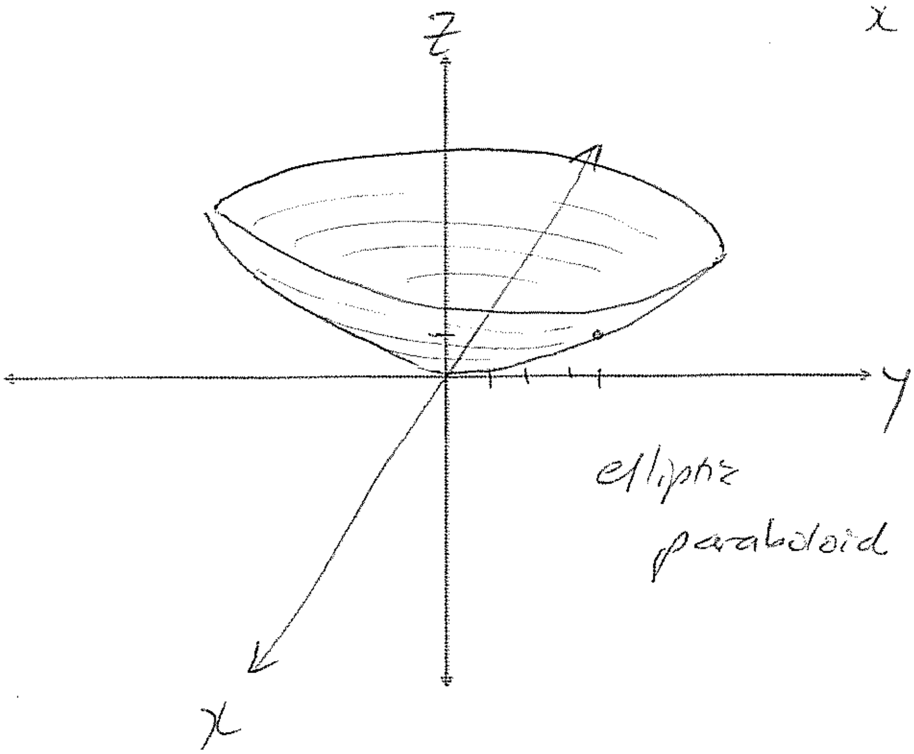
(d)

(v)



(vi)

7. (7 points) Sketch the surface given by the equation $x^2 + y^2 - 16z = 0$. You will need to add an x-axis to the graph below.



$x^2 + y^2 = 16z$

rotational surface with $y = 4\sqrt{z}$ as generating curve

$y^2 = 16z$

$\frac{y^2}{16} = z$

Extra Credit(1 point) Let $\mathbf{v} = \langle \mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3} \rangle$ and c be a scalar. Prove that $||c\mathbf{v}|| = |\mathbf{c}|\,||\mathbf{v}||$.

$$||cv|| = \sqrt{(cv_1)^2 + (cv_2)^2 + (cv_3)^2}$$
$$= \sqrt{c^2(v_1^2 + v_2^2 + v_3^2}$$
$$= |c|\sqrt{v_1^2 + v_2^2 + v_3^2}$$
$$= |c|\,||v||$$

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FORMULAS

- $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{||\mathbf{u}||\,||\mathbf{v}||}$
- $D = \dfrac{|\overrightarrow{PQ}\cdot\mathbf{n}|}{||\mathbf{n}||}$
- $x = x_1 + at,\ y = y_1 + bt,\ z = z_1 + ct$
- $\left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$
- $D = \dfrac{||\overrightarrow{PQ}\times\mathbf{u}||}{||\mathbf{u}||}$
- $W = \mathbf{F}\cdot\overrightarrow{PQ}$
- $||\mathbf{u}\times\mathbf{v}|| = ||\mathbf{u}||\,||\mathbf{v}||\sin(\theta)$
- $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$
- $ax + by + cz + d = 0$
- $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$
- $z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$
- $z = \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}$
- $\dfrac{z^2}{c^2} - \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
- $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$
- $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$
- $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 0$
- $y^2 + z^2 = [r(x)]^2$
- $r^2 = \rho^2\sin^2(\phi), \qquad \theta = \theta, \qquad z = \rho\cos(\phi)$
- $\rho = \sqrt{r^2+z^2}, \qquad \theta = \theta, \qquad \phi = \arccos\left(\dfrac{z}{\sqrt{r^2+z^2}}\right)$
- $x = \rho\sin(\phi)\cos(\theta), \qquad y = \rho\sin(\phi)\sin(\theta), \qquad z = \rho\cos(\phi)$
- $\rho^2 = x^2+y^2+z^2, \qquad \tan(\theta) = \dfrac{y}{x}, \qquad \phi = \arccos\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right)$
- $r^2 = x^2+y^2 \qquad \tan(\theta) = \dfrac{y}{x} \qquad z = z$
- $x = r\cos(\theta) \qquad y = r\sin(\theta) \qquad z = z$